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$$x^2 + y^2 = (a_1 a_2 a_3 \cdots a_m)^n,$$

has, in Legendre's notation, $E((n+1)^m/2)$ solutions. Show that in 2^{m-1} of these solutions x and y are relatively prime.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

445. Proposed by S. A. JOFFE, New York City.

Sum the series

$$\binom{cn}{a} - \binom{a}{1} \binom{cn-c}{a} + \binom{a}{2} \binom{cn-2c}{a} - \cdots + (-1)^a \binom{cn-ca}{a}.$$

I. SOLUTION BY THE PROPOSER.

This is a generalization of the series forming the first member of equation (2) in the Proposer's solution of problem No. 424 (*June 1915 issue*, p. 205), obtained by multiplying the upper indices $n, n-1, n-2, \dots$ by the constant factor c . Following the method employed in that solution, we find that the given series equals

$$\Delta^a \binom{cn-ca}{a},$$

the finite differences being taken with respect to n .

Now

$$\Delta_x \binom{cx}{a} = \binom{cx+1}{a} - \binom{cx}{a},$$

the second member of which may be written in the following form:

$$\begin{aligned} \binom{cx+c}{a} - \binom{cx+c-1}{a} + \binom{cx+c-1}{a} - \binom{cx+c-2}{a} + \binom{cx+c-2}{a} - \cdots \\ + \binom{cx+1}{a} - \binom{cx}{a}, \end{aligned}$$

since all these terms, except the first and last, alternately cancel each other. Combining the terms in pairs and noticing that

$$\binom{cx+c}{a} - \binom{cx+c-1}{a} = \binom{cx+c-1}{a-1}, \quad \binom{cx+c-1}{a} - \binom{cx+c-2}{a} = \binom{cx+c-2}{a-1},$$

etc., we have

$$\Delta_x \binom{cx}{a} = \binom{cx+c-1}{a-1} + \binom{cx+c-2}{a-1} + \cdots + \binom{cx+1}{a-1} + \binom{cx}{a-1},$$

which means that the first difference, taken with respect to x , of the binomial coefficient $\binom{cx}{a}$ having for its lower index a , equals the sum of c binomial coefficients, each having for its lower index $a-1$.

In the same manner, the second difference $\Delta_x^2 \binom{cx}{a}$ may be expressed as the sum of $c \cdot c = c^2$ binomial coefficients, each having $a-2$ for its lower index; and continuing this process, we find that the a th difference $\Delta_x^a \binom{cx}{a}$ may be expressed as the sum of c^a binomial coefficients, each having for its lower index $a-a=0$ and hence each equal to 1. In other words,

$$\Delta_x^a \binom{cx}{a} = c^a,$$

and, similarly,

$$\Delta_n^a \binom{cn-a}{a} = c^a.$$

We thus arrive at the result:

$$\binom{cn}{a} - \binom{a}{1} \binom{cn-c}{a} + \binom{a}{2} \binom{cn-2c}{a} - \cdots + (-1)^a \binom{cn-ca}{a} = c^a.$$

II. SOLUTION BY NORMAN ANNING, Chilliwack, B. C.

Define operators E and Δ as follows:

$$E^h f(n) = f(n+h), \quad \Delta f(n) = f(n+1) - f(n) = (E-1)f(n).$$

Then the given expression,

$$\begin{aligned} \binom{cn}{a} - \binom{a}{1} \binom{cn-c}{a} + \binom{a}{2} \binom{cn-2c}{a} - \cdots + (-1)^a \binom{cn-ca}{a} \\ = (1 - E^{-1})^a \binom{cn}{a} = (\Delta E^{-1})^a \binom{cn}{a} = \Delta^a E^{-a} \binom{cn}{a} = \Delta^a \binom{cn-ca}{a}. \end{aligned}$$

Now

$$\binom{cn-ca}{a} = \frac{1}{a!} \{c^a n^a + \text{terms in } n \text{ of degree lower than } a\}.$$

Hence,

$$\Delta^a \binom{cn-ca}{a} = \frac{1}{a!} \{c^a \cdot a! + 0\} = c^a.$$

446. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Solve the equations

$$x^2(y-z) = l^2(m-n), \quad y^2(z-x) = m^2(n-l), \quad z^2(x-y) = n^2(l-m).$$

SOLUTION BY H. S. UHLER, Yale University.

An equation involving only one variable x may be obtained in the following manner. Equating the expressions for z derived from the first two given equations we find

$$y - l^2(m-n)/x^2 = x + m^2(n-l)/y^2$$

or

$$x^2 y^2 - x^2 y^2 + m^2(n-l)x^2 + l^2(m-n)y^2 = 0. \quad (1)$$

Substituting the value of z from the first in the third given equation and reducing, we obtain

$$x^5 y^2 - x^4 y^2 - n^2(l-m)x^4 - 2l^2(m-n)x^2 y + 2l^2(m-n)x^2 y^2 + l^4(m-n)^2 x - l^4(m-n)^2 y = 0. \quad (2)$$

Multiplying (1) by x^2 and subtracting (2) from the product gives, after removing the factor $m-n$,

$$(lm-mn+nl)x^4 - 2l^2 x^2 y + l^2 x^2 y^2 + l^4(m-n)x - l^4(m-n)y = 0. \quad (3)$$

Multiplying (1) by l^2 , (3) by y , adding, and dividing by x ,

$$(lm-mn+nl)x^2 y - l^2 x^2 y^2 + l^2 m^2(n-l)x + l^4(m-n)y = 0. \quad (4)$$

Adding (3) and (4) and dividing by x ,

$$(lm-mn+nl)x^3 + (lm-mn+nl-2l^2)x^2 y + l^2[l^2(m-n) + m^2(n-l)] = 0. \quad (5)$$

Substituting the expression for y from (5) in (4) and removing the factors $(l-m)(l-n)$ we derive the following quadratic in x^2 , namely

$$(lm-mn+nl)^2 x^6 - 2l^4[m^2(l-n) + n^2(l-m)]x^3 - l^6(mn-nl+lm)(nl-lm+mn) = 0.$$

Consequently

$$x^3 = l^3$$

and

$$x^3 = \frac{l^3[l^2(m-n)^2 - m^2 n^2]}{(lm-mn+nl)^2}.$$